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分散型修正項をもつ双曲型特異摂動の漸近解の構成について

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1 Introduction

We consider Cauchy problems for a linear strictly hyperbolic equation of order l with a small parameter $\epsilon \in (0, \epsilon_0]$:

$$\left((i\epsilon)^{l-m} L(t, x, D_t, D_x; \epsilon) + M(t, x, D_t, D_x; \epsilon)\right) u(t, x; \epsilon) = f(t, x; \epsilon) \tag{1}$$

for $(t, x) \in (0, T) \times \boldsymbol{R}_x^d$,

$$D_t^j u(0, x; \epsilon) = g_j(x; \epsilon) \qquad j = 0, 1, 2, \ldots, l-1 \tag{2}$$

where L and M are linear strictly hyperbolic operators of order l and m ($l = m+1$ or $m+2$) with C^∞ bounded derivatives with respect to $(t, x, \epsilon) \in [0, \infty) \times \boldsymbol{R}^d \times [0, \epsilon_0]$.

The aim of this paper is to give C^∞ asymptotic expansions of solutions to singularly perturbed Cauchy problems of this type. This is a revisit of problems treated in [8].

We postulate that the solution has an expansion

$$
\begin{aligned}
(3) \qquad u(t,x;\epsilon) &\sim v(t,x;\epsilon) + w(t,x;\epsilon), \\
(4) \qquad v(t,x;\epsilon) &= \sum_{n=0}^{\infty} \epsilon^n v_n(t,x) \quad \text{(regular part)}, \\
(5) \qquad w(t,x;\epsilon) &= \sum_{n=m}^{\infty} \epsilon^n w_n(t,x;\epsilon) \quad \text{(singular part)}
\end{aligned}
$$

where v and w mean formal sums such that

$$
\begin{aligned}
(6) \qquad Pv &\sim f \\
(7) \qquad Pw &\sim 0 \\
(8) \qquad D_t^j(v+w)\Big|_{t=0} &\sim g_j, \quad j = 0,1,2,\ldots,l-1.
\end{aligned}
$$

We investigated in [9] a priori L^2 and higher order Sobolev norm estimates of the solution to (1) and (2) under various separation conditions of characteristic roots of L and M. In [10], we dealt with the case where the singular part, that is, the correction terms (5) associated with (4) were of dissipative type (exponential decay as ϵ tends to 0). In this paper, we treat the case where the the correction terms are dispersive (highly oscillating as ϵ tends to 0). They are described by oscillating functions locally and by Maslov's canonical operators globally. The estimates of the remainder terms of asymptotic expansions are given by a priori estimates in [9].

In view point of propagation of waves, the regular part of the solution is governed by the principal part of M (the subcharacteristic wave in [11]). The singular part is governed by ϵ -principal part of $(i\epsilon)^{l-m}L + M$. In contrast with the propagation of singularity of the solution u, the principal part of L is not *principal* to determine the quantitative propagation of the singularly perturbed wave.

2 A priori estimates

We consider two operators L and M :

$$L(t,x,D_t,D_x;\epsilon) = D_t^l + \sum_{j=1}^{l} L_j(t,x,D_x;\epsilon)D_t^{l-j} \tag{9}$$

$$M(t,x,D_t,D_x;\epsilon) = m_0(t,x,D_x;\epsilon)D_t^m + \sum_{j=1}^{m} M_j(t,x,D_x;\epsilon)D_t^{m-j} \tag{10}$$

with their principal symbols

$$l(t,x,\tau,\xi;\epsilon) = \tau^l + \sum_{j=1}^{l} l_j(t,x,\xi;\epsilon)\tau^{l-j} \tag{11}$$

$$m(t,x,\tau,\xi;\epsilon) = m_0(t,x,\xi;\epsilon)\tau^m + \sum_{j=1}^{m} m_j(t,x,\xi;\epsilon)\tau^{m-j}. \tag{12}$$

We assume the following assumptions:

(H0) Regular Hyperbolicity of L: $l(t,x,\tau,\xi;\epsilon)$ has the decomposition

$$l(t,x,\tau,\xi;\epsilon) = \prod_{j=1}^{l}(\tau - \varphi_j(t,x,\xi;\epsilon)) \tag{13}$$

where $\varphi_j(t,x,\xi;\epsilon)$ are real distinct elements such that

$$\varphi_1(t,x,\xi;\epsilon) < \varphi_2(t,x,\xi;\epsilon) < \cdots < \varphi_l(t,x,\xi;\epsilon) \quad \text{uniformly:} \tag{14}$$

that is,$\varphi_{j+1}(t,x,\xi;\epsilon) - \varphi_j(t,x,\xi;\epsilon)$ is uniformly positive for $j = 1,\cdots,l-1$.

(H1) Regular Hyperbolicity of M: $m(t,x,\tau,\xi;\epsilon)$ has the decomposition

$$m(t,x,\tau,\xi;\epsilon) = m_0(t,x,\xi;\epsilon)\prod_{j=1}^{m}(\tau - \psi_j(t,x,\xi;\epsilon)) \tag{15}$$

where $\psi_j(t,x,\xi;\epsilon)$ are real distinct elements such that

$$\psi_1(t,x,\xi;\epsilon) < \psi_2(t,x,\xi;\epsilon) < \ldots < \psi_m(t,x,\xi;\epsilon) \quad \text{uniformly.} \tag{16}$$

When $l = m+1$, we assume the following assumptions (H2) and (S0).

(H2): $m_0(t,x;\epsilon)$ is pure-imaginary and uniformly away from 0, that is,

$$\Re m_0(t,x;\epsilon) = 0 \quad \text{and} \quad |\Im m_0(t,x;\epsilon)| \geq \delta > 0,$$

(S0): $\{\psi_i\}$ separates $\{\varphi_j\}$ uniformly, that is,

$$\varphi_1 < \psi_1 < \varphi_2 < \cdots < \psi_m < \varphi_{m+1} \quad \text{uniformly.}$$

Remark 1 Since L and M are differential operators, the conditions (H2) and (S0) are equivalent to $\left(\mathrm{WS}^{\pm}\right)$ and $\left(\mathrm{S}^{\pm}\right)$ in [9].

Remark 2 In [10], we assumed (H0), (H1), (S0) and

(E1): the uniformly strong ellipticity of m_0, that is,

$$\Re m_0(t, x; \epsilon) \geq \delta > 0.$$

We quote from [9]

Theorem 2.1 *Under the assumptions* (H0),(H1),(H2) *and* (S0), *for any natural number p, there exist $C > 0$ and γ_0 such that for any positive $\epsilon \leq \epsilon_0$, any $\gamma \geq \gamma_0$ and for any $u(t) \in C^\infty\left([0,T]; C_0^\infty(\boldsymbol{R}_x^d)\right)$ we have*

$$\begin{aligned}
(17) \qquad & C\left\{\frac{1}{\gamma}\int_0^T e^{-2\gamma t}\frac{1}{\epsilon}\sum_{j=0}^{p}\left(\epsilon^2\gamma\right)^j \| D^j f(t) \|^2\, dt + \| D^{m-1}u(0) \|_{1/2}^2\right. \\
+ & \ \gamma^p\left(\epsilon\sum_{j=0}^{p}\epsilon^{2j} \| D^m u(0) \|_j^2 + \sum_{j=1}^{p}\epsilon^{2j} \| D^m u(0) \|_{j-1/2}^2\right. \\
+ & \left.\left.\ \epsilon\sum_{j=0}^{p-1}\epsilon^{2j} \| D^j f(0) \|^2 + \sum_{j=1}^{p-1}\epsilon^{2j} \| D^{j-1} f(0) \|_{1/2}^2\right)\right\} \\
\geq & \ \gamma\int_0^T e^{-2\gamma t}\sum_{j=0}^{p}\left(\epsilon^2\gamma\right)^j\left(\epsilon \| D^{m+j}u(t) \|^2 + \| D^{m+j-1}u(t) \|_{1/2}^2\right) dt \\
+ & \ e^{-2\gamma T}\sum_{j=0}^{p}\left(\epsilon^2\gamma\right)^j\left(\epsilon \| D^{m+j}u(T) \|^2 + \| D^{m+j-1}u(T) \|_{1/2}^2\right).
\end{aligned}$$

When $l = m + 2$, we assume (H0), (H1) and the following assumptions (WS) and (P):

(WS): $\{\psi_i\}$ weakly separates $\{\varphi_j\}$ uniformly, that is,

$$\varphi_1 < \{\psi_1, \varphi_2\} < \cdots < \{\psi_{m+1}, \varphi_m\} < \varphi_{m+2} \quad \text{uniformly,}$$

where $\{a,b\} < \{c,d\}$ means $\max\{a,b\} < \min\{c,d\}$.

(P): $m_0(t,x;\epsilon)$ is real and uniformly positive, that is,

$$\Im m_0(t,x;\epsilon) = 0, \quad \text{and} \quad m_0(t,x:\epsilon) \geq \delta > 0.$$

We quote from [10],

Theorem 2.2 *Under the assumptions* (H0), (H1), (P) *and* (WS), *for any natural number p, there exist positive constant C and γ_0 such that any $\epsilon \in (0,\epsilon_0]$, for any $\gamma \geq \gamma_0$, for any any $u(t) \in C^\infty\left([0,T];C_0^\infty\left(\boldsymbol{R}_x^d\right)\right)$ we have*

$$C\left\{\frac{1}{\gamma}\int_0^T e^{-2\gamma t}\frac{1}{\epsilon^2}\sum_{j=0}^{p}\left(\epsilon^2\gamma\right)^j \| D^j f(t) \|^2\, dt + \gamma^p\|D^m u(0)\|^2\right. \tag{18}$$

$$\left. + \quad \gamma^p\left(\epsilon\sum_{j=0}^{p}\epsilon^{2j+2} \| D^{m+1}u(0) \|_j^2 + \sum_{j=0}^{p-1}\epsilon^{2j} \| D^j f(0) \|^2\right)\right\}$$

$$\geq \gamma\int_0^T e^{-2\gamma t}\sum_{j=0}^{p}\left(\epsilon^2\gamma\right)^j\left(\epsilon^2 \| D^{m+j+1}u(t) \|^2 + \| D^{m+j}u(t) \|^2\right) dt$$

$$+ \quad e^{-2\gamma T}\sum_{j=0}^{p}\left(\epsilon^2\gamma\right)^j\left(\epsilon^2 \| D^{m+j+1}u(T) \|^2 + \| D^{m+j}u(T) \|^2\right).$$

3 Singular characteristic roots.

3.1 degeneration of order 1.

Let $l = m+1$. We define ϵ-principal symbol

$$p(t,x,\tau,\xi;\epsilon) = il(t,x,\tau,\xi;\epsilon) + m(t,x,\tau,\xi;\epsilon).$$

We denote the roots of $p(\tau) = 0$ by $\tau_j(t,x,\xi;\epsilon)$'s.

Proposition 3.1 *We assume* (H0),(H1),(H2) *and* (S0). *Then, τ_j's are real and uniformly distinct, that is,*

$$\tau_1 < \tau_2 < \cdots < \tau_{m+1}.$$

Moreover, if

$$\Im m_0(t,x;\epsilon) \geq \delta > 0, \tag{19}$$

the least root $\tau_1(t,x,\xi;\epsilon)$ *belongs to the nonhomogeneous smooth symbol class* S^1 *and* $\tau_1(t,x,0;\epsilon) = -\Im m_0(t,x;\epsilon)$.

If

$$-\Im m_0(t,x;\epsilon) \geq \delta > 0, \tag{20}$$

the greatest root $\tau_{m+1}(t,x,\xi;\epsilon)$ *belongs to the nonhomogeneous smooth symbol class* S^1 *and* $\tau_{m+1}(t,x,0;\epsilon) = -\Im m_0(t,x;\epsilon)$.

Remark When the condition (19) holds, we have

$$\tau_1 < \varphi_1 < \psi_1 < \cdots < \psi_m < \tau_{m+1} < \varphi_{m+1}.$$

We call τ_1 the singular root. Aternatively, τ_{m+1} is the singular one, when the condition (20) holds.

We denote for simplicity, $p(t,x,\tau,\xi;0)$ by p, $\tau_1(t,x,\xi;0)$ by τ_1 and so on. We consider a Hamiltonian system for $(t(\sigma),x(\sigma),\tau(\sigma),\xi(\sigma))$:

$$\begin{cases} \dfrac{dt}{d\sigma} = \dfrac{\partial p}{\partial \tau}, & \dfrac{dx_j}{d\sigma} = \dfrac{\partial p}{\partial \xi}, \qquad j=1,2,\cdots,d, \\[2ex] \dfrac{d\tau}{d\sigma} = -\dfrac{\partial p}{\partial t}, & \dfrac{d\xi_j}{d\sigma} = -\dfrac{\partial p}{\partial x_j}, \qquad j=1,2,\cdots,d, \end{cases} \tag{21}$$

with Cauchy data

$$\begin{cases} t(0) = 0, & x_j(0) = y_j, \qquad j=1,2,\cdots,d, \\ \tau(0) = \tau_1(0,y,0;0), & \xi_j(0) = 0, \qquad j=1,2,\cdots,d. \end{cases} \tag{22}$$

Proposition 3.2 (Fedoryuk[2]) *The family of* $t = t(\sigma, y)$, $x = x(\sigma, y)$, $\tau = \tau_1(t(\sigma, y), x(\sigma, y), \xi(\sigma, y))$, $\xi = \xi(\sigma, y)$ *is a unique solution to (21) and (22), if and only if* $\tilde{x}(t, y) = x(\sigma(t, y), y)$ *and* $\tilde{\xi}(t, y) = \xi(\sigma(t, y), y)$ *satisfy the Hamiltonian system*

$$\left\{\begin{array}{ll} \dfrac{d\tilde{x}_j}{d\sigma} = -\dfrac{\partial \tau_1}{\partial \xi}, & j = 1, 2, \cdots, d, \\[2ex] \dfrac{d\tilde{\xi}_j}{d\sigma} = \dfrac{\partial \tau_1}{\partial x_j}, & j = 1, 2, \cdots, d, \end{array}\right. \tag{23}$$

and Cauchy data

$$\tilde{x}(0) = y, \quad \tilde{\xi}(0) = 0. \tag{24}$$

Proposition 3.3 *We assume the above assumptions.*

(i) We have a unique system of solutions $\{\tilde{x}_i(t, y)\}$ *and* $\{\tilde{\xi}_i(t, y)\}$ *to (23) and (24) for all non-negtive t. There exists a positive constant M such that for any nonnegative t*

$$\sup_y \left|\tilde{x}_i(t, y) - y_i\right| \leq Mt \qquad i = 1, 2, \cdots, d,$$

$$\sup_y \left|\tilde{\xi}_i(t, y)\right| \leq e^{Mt} - 1, \qquad i = 1, 2, \cdots, d.$$

(ii) There exist positive constants T_0, δ, *such that*

$$\left|\det\left(\frac{\partial \tilde{x}_i}{\partial y_a}(t, y)\right)\right| \geq \delta > 0 \quad (t, y) \in [0, T_0] \times \boldsymbol{R}^d.$$

3.2 degeneration of order 2.

Let $l = m + 2$. We define ϵ-principal symbol

$$p(t, x, \tau, \xi; \epsilon) = -l(t, x, \tau, \xi; \epsilon) + m(t, x, \tau, \xi; \epsilon).$$

We denote the roots of $p(\tau) = 0$ by $\tau_j(t, x, \xi; \epsilon)$'s.

Proposition 3.4 *We assume* (H0),(H1),(P) *and* (WS). *Then,* τ_j*'s are real and uniformly distinct, that is,*

$$\tau_1 < \tau_2 < \cdots < \tau_{m+2}.$$

Moreover, the least root $\tau_1(t,x,\xi;\epsilon)$ *and the greatest root* $\tau_{m+2}(t,x,\xi;\epsilon)$ *are inhomogeneous symbols in* S^1. *They satisfy* $\tau_1(t,x,0;\epsilon) = -\sqrt{m_0(t,x;\epsilon)}$ *and* $\tau_{m+2}(t,x,0;\epsilon) = \sqrt{m_0(t,x;\epsilon)}$

Remark. We have for $j = 2,3,\cdots,m+1$,

$$\tau_1 < \varphi_1 < \min\{\varphi_j, \psi_{j-1}\} < \tau_j < \max\{\varphi_j, \psi_{j-1}\} < \varphi_{m+2} < \tau_{m+2}.$$

We call τ_1 and τ_{m+2} singular root.

We consider the Hamiltonian systems of the same type as in the previous subsection, except one condition in the Cauchy data,

$$\tag{25} \begin{aligned} \tau|_{\sigma=0} &= \tau_i(0,y,0) \quad \text{for} \quad i=1 \quad \text{or} \quad m+2 \\ &= \pm\sqrt{m_0(0,x;0)}. \end{aligned}$$

We obtain the solutions $(t^*(\sigma), x^*(\sigma), \xi^*(\sigma))$ and $\left(\tilde{x}^*(t,y), \tilde{\xi}^*(t,y)\right)$, where $* = \pm$ according to the signature of the Cauchy data (25).

4 Canocical operators of Maslov

We refer details to Maslov and Fedoriuk [6] and other references [2], [1], [3], [7] related to Maslov [5].

Let Λ^{d+1} be the flow-out of $\boldsymbol{R}^{d}_{x}\times\{0\}\subset\boldsymbol{R}^{d}_{x}\oplus\boldsymbol{R}^{d}_{\xi}$, by the trajectory (23) for $t\in[0,\infty)$. That is,

$$\Lambda^{d+1} = \left\{(t,x,\tau,\xi)\in\boldsymbol{R}^{d+1}_{t,x}\oplus\boldsymbol{R}^{d+1}_{\tau,\xi};0\le t<\infty, x=\tilde{x}(t,y),\right. \tag{26}$$

$$\left.\tau(t)=\tau_1(t,\tilde{x}(t,y),\tilde{\xi}(t,y)),\xi=\tilde{\xi}(t,y)\right\}. \tag{27}$$

Proposition 4.1 (Fedoryuk [2]) *(i) Λ^{d+1} is a $(d+1)$-dimensional simply connected nonhomogeneous Lagrangian C^∞ manifold with boundary*

$$\begin{aligned}\Lambda' &= \left\{(0,y,\tau_1(0,y,0);y\in\boldsymbol{R}^{d}\right\}\\ &\cong \boldsymbol{R}^{d}.\end{aligned}$$

(ii) The variable t can be always in a set of local coordinates of any point of Λ^{d+1}.

(iii) The projection of the restricted part $\Lambda^{d+1}\big|_{[0,T_0]}$ onto $\boldsymbol{R}^{d+1}_{t,x}\big|_{[0,T_0]}$ along $\boldsymbol{R}^{d+1}_{\tau,\xi}$ is a diffeomorphism.

Λ^{d+1} has a global system of coordinates $(t,y)\leftrightarrow\lambda\in\Lambda^{d+1}$. This defines a volume element $d\sigma(\lambda(t,y)) = dtdy$ on Λ^{d+1}, which is invariant by the Hamiltonian flow. We choose a locally finite covering of canonical charts $\{\Lambda_j\}_{j=0}^{\infty}$ of Λ^{d+1} where $\Lambda_0=\Lambda^{d+1}\big|_{[0,T_0]}$. Λ_j has a canonical coordinates $\lambda_j(t,x_{I(j)},\xi_{\overline{I}(j)})$ where $I(j)\cup\overline{I}(j) = \{1,2,\cdots,d\}$ and $I(j)\cap\overline{I}(j)=\emptyset$. We associate a C^∞ partition of unity $\left\{e_j(t,x_{I(j)},\xi_{\overline{I}(j)})\right\}$ with $\{\Lambda_j\}_{j=0}^{\infty}$.

For $h\in C_0^\infty(\Lambda)$, we define the global canonical operator K_Λ by

$$(K_\Lambda h)(t,x)=\sum_{j=1}^{\infty}K_{\Lambda_j}(e_jh)(t,x)$$

, where K_{Λ_j} is the precanonical operator (See [2], [6]).

In the same way, the global canonical operators K_{Λ^*}, where $*=\pm$, are defined.

5 Formal construction of asymptotic solutions.

For any $n \in N$, we have the Taylor expanion of L:

$$L(t,x,D_t,D_x;\epsilon) = \sum_{n=0}^{N} \epsilon^n L^{(n)}(t,x,D_t,D_x) + R_{N+1}(L;\epsilon),$$

where $L(t,x,D_t,D_x;\epsilon)$ and $R_{N+1}(L;\epsilon)$ are differential operators of order $m+1$. We have also

$$M(t,x,D_t,D_x;\epsilon) = \sum_{n=0}^{N} \epsilon^n M^{(n)}(t,x,D_t,D_x) + R_{N+1}(M;\epsilon),$$

where $M(t,x,D_t,D_x;\epsilon)$ and $R_{N+1}(M;\epsilon)$ are differential operators of order m.

5.1 degeneration of order 1.

We consider $P = \epsilon L + M$. The problem is

$$\begin{cases} Pu = f, \\ D_t^j u(0,x;\epsilon) = g_j(x;\epsilon), \qquad 0 \le j \le m. \end{cases}$$

We introduce

$$\tilde{P}(t,x,\epsilon D_t,\epsilon D_x;\epsilon) = \epsilon^m P(t,x,D_t,D_x;\epsilon).$$

We assume for the singular part

$$w \sim \sum_{n=m}^{\infty} \epsilon^n w_n = \sum_{n=m}^{\infty} \epsilon^n K_\Lambda h_n.$$

The equations (6), (7) and (8) expanded with respect to ϵ determine successively v_n's and h_n's.

5.2 degeneration of order 2.

We consider $P = (i\epsilon)^2 L + M$. The problem is

$$\begin{cases} Pu = f, \\ D_t^j u(0, x; \epsilon) = g_j(x; \epsilon), \qquad 0 \leq j \leq m+1. \end{cases}$$

We introduce

$$\tilde{P}(t, x, \epsilon D_t, \epsilon D_x; \epsilon) = \epsilon^m P(t, x, D_t, D_x; \epsilon).$$

We assume for the singular part

$$w \sim \sum_{n=m}^{\infty} \epsilon^n w_n = \sum_{\substack{n=m \\ *=\pm}}^{\infty} \epsilon^n K_{\Lambda *} h_n^*.$$

6 Remainder estimates of asymptotic solutions.

6.1 degeneration of order 1.

We define the partial sum by

$$u_N(t, x; \epsilon) = \sum_{n=0}^{N} \epsilon^n v_n(t, x) + \sum_{n=m}^{N+m} \epsilon^n K_\Lambda h_n(t, x; \epsilon)$$

and its remainder term by

$$R_{N+1}(u; \epsilon) = u(t, x; \epsilon) - u_N(t, x; \epsilon).$$

Our main result is

Theorem 6.1 *Let T be a fixed positive number. Let $f \in C^\infty([0, T]; C_0^\infty(\boldsymbol{R}^d))$ and $g_j \in C_0^\infty(\boldsymbol{R}^d)$. There exists a positive constant C such that for any $\epsilon \in (0, \epsilon_0]$,*

$$\begin{aligned} (C\epsilon)^{2(N+1)-2m} \geq & \int_0^T \sum_{j=0}^{p} \epsilon^{2j} \left(\epsilon \parallel D^{m+j} R_{N+1}(u; \epsilon)(t) \parallel^2 + \parallel D^{m+j-1} R_{N+1}(u; \epsilon)(t) \parallel_{1/2}^2 \right) dt \\ & + \sum_{j=0}^{p} \epsilon^{2j} \left(\epsilon \parallel D^{m+j} R_{N+1}(u; \epsilon)(T) \parallel^2 + \parallel D^{m+j-1} R_{N+1}(u; \epsilon)(T) \parallel_{1/2}^2 \right). \end{aligned}$$

Corollary *For any $k, N_0 \in \boldsymbol{N}$ and positive T, there exist $N_1 \in \boldsymbol{N}$ such that for any $N \geq N_1$ there exists a positive constant C_{N,N_0} independent from ϵ such that*

$$\sup_{\substack{0\leq t\leq T\\ x\in \boldsymbol{R}^d}} \sum_{j+|\alpha|\leq k} |D_t^j D_x^\alpha R_{N+1}(u;\epsilon)(t,x)| \leq C_{N,N_0}\epsilon^{N_0}.$$

Remark The constants C and C_{N,N_0} depend on the support of f and g_j's.

6.2 degeneration of order 2.

We define the partial sum by

$$u_N(t,x;\epsilon) = \sum_{n=0}^{N} \epsilon^n v_n(t,x) + \sum_{\substack{n=m\\ *=\pm}}^{N+m} \epsilon^n K_{\Lambda^*} h_n^*(t,x;\epsilon)$$

and its remainder term by

$$R_{N+1}(u;\epsilon) = u(t,x;\epsilon) - u_N(t,x;\epsilon).$$

Our main result is

Theorem 6.2 *Let T be a fixed positive number. Let $f \in C^\infty([0,T]; C_0^\infty(\boldsymbol{R}^d))$ and $g_j \in C_0^\infty(\boldsymbol{R}^d)$. There exists a positive constant C such that for any $\epsilon \in (0,\epsilon_0]$,*

$$\begin{aligned} C\epsilon^{2(N+1)-2m} \;\geq\; & \int_0^T \sum_{j=0}^{p} \epsilon^{2j} \left(\epsilon^2 \| D^{m+j+1} R_{N+1}(u;\epsilon)(t) \|^2 + \| D^{m+j} R_{N+1}(u;\epsilon)(t) \|^2\right) dt \\ & + \sum_{j=0}^{p} \epsilon^{2j} \left(\epsilon^2 \| D^{m+j+1} R_{N+1}(u;\epsilon)(T) \|^2 + \| D^{m+j} R_{N+1}(u;\epsilon)(T) \|^2\right). \end{aligned}$$

Corollary *For any $k, N_0 \in \boldsymbol{N}$ and positive T, there exist $N_1 \in \boldsymbol{N}$ such that for any $N \geq N_1$ there exists a positive constant C_{N,N_0} independent from ϵ such that*

$$\sup_{\substack{0\leq t\leq T\\ x\in \boldsymbol{R}^d}} \sum_{j+|\alpha|\leq k} |D_t^j D_x^\alpha R_{N+1}(u;\epsilon)(t,x)| \leq C_{N,N_0}\epsilon^{N_0}.$$

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